DSP exercise

一、 Linear Prediction

the idea of linear prediction is a powerful one in signalmodeling. It's also directly connected to the use of all-pole modelsin spectrum estimation.

In the prediction problem, we are given a signal x[n] and wewant to build a system that will predict future values. A linearpredictor does this with FIR filter,

xx[n] = sum(-a(k)*x[n-k]) for k=1,2,...,P (1)

The best linear predictor will be one that minimizes an error such asleast squares. If we want xx[n] to be a "prediction" of the futurevalue, x[n+r], we minimize

E = sum|x[n+r]-xx[n]|^2 for n (2)

by choosing the predictor coefficients {a(k)}. The rang of the sum,to be specified later.

VERIFY that the minimization of E is to solve a linear equation in the matrix form as

-x=Xa

where the vector x and matrix X contain known signal values, and the result of {a(k)} define the FIR linear predictor.

二、 DTFT of complex exponential

The Fourier representation of a signal via the forward and inverse DTFT is a key part of signal analysis. The following Equation is the forward one.

DTFT(x[n])=X(exp(jw))= sum(x[n]*exp(-jwn)),

for n= -infinite to infinite

Derive that DTFT of a complex exponential, which in form as

x[n]=z^n*u[n]

where z is a complex constant and u[n] is step function,can be achieve by an IIR filter (a two order difference equation).

If |z|=r<1 and phase(z)=thita, try to develop a simple formula ,relates the bandwith (half decay or -3dB attenuation in frequency,domain) to r and thita.

三、 Notch Filter

A notch filter attempts to remove one particular frequency.Suppose that a bandlimited continuous-time signal is known to contain
a 60 Hz interference component, which we want to remove by processing
with the standard system for filtering a continuous-time signal with a discrete-time filter:

xc(t)-->A/D-->x[n]-->Discrete filter-->y[n]-->D/A-->yr(t)

Assume that the sampling freequency is 1kHz (signal is bandlimited to avoid aliasing), and the frequency response of the discrete filteris,

```
              [1-exp(-j(w-w0))][1-exp(-j(w+w0))]
H(exp(jw))= --------------------------------------------
           [1-0.9*exp(-j(w-w0))][1-0.9*exp(-j(w+w0))]
```

Use any software such as MATLAB or LabVIEW to plot the magnitude and phase of H(exp(jw)).

What the value should be chosen for w0 to eliminate the 60 Hz component?

四、 Group Delay

A convenient measure of the linearity of the phase is the group delay.The group delay of a sys tem is defined as,

```
                 -d
t(w)=grd[H(exp(jw))]= ----{arg[H(exp(jw))]}
                 dw
```

This derivative cannot be taken directly unless the phase is unwrapped to remove the 2*pi jumps. However, the phase unwrapping can be avoid by using an alternative algorithm based on the discrete-time Fourier transform property that

```
if   h[n]-->DFT-->H(exp(jw))
                        dH(exp(jw))
   then  n*h[n]-->DFT-->j*-------------
                            dw
```

Express H(exp(jw)) in polar form as H(exp(jw))=A(w)*exp(j*thita(w)),where A(w) is real and prove the following property:

```
  -dthita(w)             j*[dH(exp(jw))/dw]
 ---------- ----- = Re{-----------------------------}
      dw                    H(exp(jw))
```

说明：

这是我最近在看的一本 DSP exersice book 中的题目, 我挑了一些比较有趣的,属于同一个 topic 的,综合起来作为一题.

这本 exercise book 是许多美国大学 DSP course 的教师参考书,由六个在 DSP 界顶尖的教授编写的.给我的感觉是对"深入浅出"的最好注解,绝对比偶们现在使用的教科书要好 n 倍.

贴题目的主要目的是督促自己每天都不要偷懒:)另一方面大家都做做也没什么坏处.题目真的很简单,难的地方我都忽略了,基本上本科生都能做,给复旦 BBS 增加一点学术气氛也不错:) 顺便说一下答案.

第一题对式(2)两边求微分就可以得到一个线性方程组.这是比较严格的解法,不严格的话,式(1)本身就是一个线性方程组.

第二题较麻烦一些,因为 DTFT of complex exponential 是一个等比数列的无限累加,当 r<1 时正好收敛成一个二阶 IIR 的频响.写出频响公式,再对这个解析的频响公式求解一下就可以得到带宽.

*第三题是一个梳状滤波器,画一下图就知道 w0 就是要抑制的频率,所以 w0=(60/1000)*2*pi.*

第四题最简单,把微分写出来就是了.